

Seminar III

Problem 1.

Paul is an entrepreneur with a business idea he wants to develop: buying a property near the Geilo skiing resort and turning it into an area with mountain cabins, in Norwegian a *hytteby*.

The project is risky. In particular, we assume that there is probability p that it is successful and a corresponding probability $(1 - p)$ that it fails. A success means that an investment of an amount I returns RI . Failure means a zero return on the project.

The probability p of success depends on Paul's own efforts in making it a success. In particular, if Paul works hard, there will be a probability $p = p_H$ of success, while if he does not work hard, this probability is $p = p_L$, where $0 < p_L < p_H < 1$, and $p_H R > 1 > p_L R$. By working hard, he will, however, suffer a loss BI proportional to the size of the project, where $0 < B < 1 - p_L R$.

Paul has available own funds of size A for this project. If he wants to invest more than A , he will need funding from outside investors. We assume that the capital market is competitive, and that limited liability prevails. Because efforts are not observable, it is not possible to contract upon effort.

a)

- i. Define the concept of *borrowing capacity*, or *debt capacity*, and find an expression for it in the case of Paul described above. Discuss how the borrowing capacity is affected by the extent of the moral-hazard problem.
- ii. What would change if, instead of the above constant-returns-to-scale investment technology, the project featured decreasing returns to scale?

- iii. Also, find an expression for Paul's *shadow value of equity*, and discuss how it is affected by the extent of the moral-hazard problem.
- b) Due to difficulties with getting the funding he wants, Paul seeks the advice of a friend, who recommends splitting the development in two, with a first-phase project whose returns RI_1 is obtained from an investment I_1 , in case of success, before the decision to launch phase two, with returns RI_2 obtained from an investment I_2 in case of success. Each of the two phases have the same success probabilities and the same moral-hazard problem as detailed above. Let us also assume that the two projects are statistically independent, and that long-term contracts are not available.
- i. Do you agree with Paul's friend that this sequential development could help on the funding?
 - ii. What would change if long-term contracts were available?
 - iii. How does the result hinge on the projects being statistically independent? To what extent is it reasonable to assume imperfect correlation in the particular case of Paul's property development?
- c) Paul realizes that there is a risk for a cost over-run in the project. In particular, there is a need, before the project is completed, for a reinvestment equal to ρI , where I is the initial investment, and ρ is distributed according to the probability distribution $F(\rho)$ on $[0, \infty)$, with density $f(\rho)$. The moral-hazard problem, in case of a reinvestment and completion of the project, is as detailed above.
- i. Discuss how this need for intermediate funds, in order to complete the project, can be dealt with in the initial contract. Explain, in particular, how the risk of a cost over-run calls for a smaller project than otherwise called for.
- d) Paul finds out that the liquidity problem raised by the prospects of a cost over-run could be mitigated if there would be a way to secure short-term returns from the project, which could be used to cover in part the cost over-run. In particular, he could sell part of the property before completion of the project, providing verifiable short-term returns rI , where again I is the initial investment, and the distribution of r is subject

to a second moral-hazard problem, in addition to the one affecting the success probability of the completed project: If Paul works hard on getting high short-term returns, he would suffer a loss B_0I and r would be distributed according to the probability distribution $G(r)$, with density $g(r)$. If not, then r is distributed according to the probability distribution $\tilde{G}(r)$, with density $\tilde{g}(r)$. Assume that the likelihood ratio, $l(r) = [g(r) - \tilde{g}(r)]/g(r)$, is (weakly) increasing in r . Define a contract as a pair of functions $\{\rho^*(r), \Delta(r)\}$, where $\rho^*(r)$ is the cutoff reinvestment need when short-term returns are r , such that the project is abandoned if $\rho > \rho^*(r)$, and $\Delta(r)$ is Paul's per-unit-of-investment extra rent, for each realization of r , over and above what is required by the other moral-hazard problem if the project is completed, and a per-unit-of-investment cash compensation if it is abandoned.

- i. Explain the meaning of $l'(r) \geq 0$. Also, explain why we need the restriction $\Delta(r) \geq 0$.
- ii. Explain why the equilibrium contract has the property that $\rho^*(r)$ is (weakly) increasing in r , and discuss features of the project that determine whether the variation in the cutoff ρ^* , as the level of short-term return r varies, is large or small.
- iii. In cases where r is low, the cutoff ρ^* may be so low that a credibility problem arises, leading to a scope for renegotiation of the initial contract. This is called the problem of the *soft budget constraint*. Explain the nature of the problem and discuss how the contract needs to be amended in order to cope with this problem.

Problem 2.

Review Problem 4, parts (i)-(iii), in Tirole, pp. 627-628.